On Different Points of View Towards:

A Practical Philosophy of Mathematics

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In this paper, we investigate several practical philosophies, motivating mathematicians to do mathematics. These practices are based on skills inside mathematics and methods of cognition outside mathematics. We also introduce and implement a hierarchy on these practical motivations, indicating a road towards philosophical maturity in doing mathematics.

Introduction

Mathematicians have several practical motivations doing mathematics and this serves as a background on the philosophy of math and more precisely, what it is that that is considered doing mathematics, how to learn mathematics and how to teach mathematics, how to do research and how to evaluate research, and how to understand history of mathematics, and how to motivate future research.

Some of these motivations are dominant is high schools and some dominant in undergraduate schools. Others dominant in graduate schools, research universities, and research institutions. Some others have not become yet dominant in any kind of scientific institution. For example, doing mathematics as study of historically developed concepts and doing calculations and computations is dominant in high school mathematics. Or discovering concepts and getting to know objects is dominant in undergraduate period. Many mathematics undergraduates keep on doing math with the same motivation as their high school period of doing mathematics. In graduate school students are motivated to regard doing mathematics as a theorem proving activity. And later after PhD they regard doing mathematics as a problem solving activity. Theorization becomes the main motivation in a research institution, paving the way for problem solvers, developing tools for future achievements. These are motivations which are originated by skills of mathematicians. There are also motivations which come from outside mathematics. For example, the cognitive approach towards math or Platonic approach towards truth in mathematics, or other approaches coming philosophy of science or philosophy of knowledge. For example, the point of view of unity of mathematics. The author acknowledges support by an Oswald Veblen grant from Institute for Advanced Study.

1) Calculations and computations

There is a group of mathematicians for whom doing mathematics is performing calculations and computations on objects and concepts which are already formed through history of mathematics. They don't look for discovering new concepts or constructing new objects to deal with. They learn about predetermined concepts via calculation and computation and teach the skill of computation to their pupils. Performing research for such a mathematician means relating existing concepts and objects by new computations determining some in terms of the others. When such a mathematician evaluates research performed by another mathematician, he asks himself, what are the new computational techniques introduced in this work which were not known before, or asks which new invariant is computed. History of mathematics in mind of such a mathematician consists of a series of calculations and computations from which sometimes new concepts and new objects are originated from. If one asks about future of mathematics from such mathematicians, they answer in terms of what new computational methods are needed, or which invariants should be calculated, or which concepts should reappear in which calculations, and so on. Such an idea of what doing mathematics could mean is very similar to high school experience of students doing mathematics.

2) Construction of objects and discovering concepts

There is a group of mathematicians for whom doing mathematics is about dealing with important objects and concepts and trying to understand them and relate them and even discover or create them. They make new objects when they need to and define new concepts when they are motivated to. The kind of mathematics they do circles around the objects and concepts they are interested in. It is not that they never compute or calculate, but they evaluate their calculations by their contribution to understanding important objects and concepts. These mathematicians experience learning mathematics, not only by getting to know preconstructed objects and prediscovered concepts, but they learn and teach the skill of constructing new objects and discovering new concepts. When they face research of their colleagues, they ask themselves what is the contribution of this paper in the language of important objects and concepts. History of mathematics is history of evolution of mathematical objects and mathematical concepts. For example, what happened to the concepts of line through the course of history. How it was originated and to which other concepts it was transformed, and with which objects and concepts it was related. These mathematicians try to foresee the future of mathematics by the objects and concepts which are to become of central importance.

3) Proving Theorems

There are mathematicians for whom it is of most importance to prove theorems and this is how they do mathematics. They see themselves as a proof machine who conjectures about what could be true and then tries to give a rigorous proof for that to make himself certain that the conjecture holds true and he is happy even when his proof is not illuminating. Learning mathematics means learning the proofs of theorems already proven and teaching mathematics is teaching the skill of proving theorems. The concept of proof is not supposed to change by such a mathematician. Doing research means exploring what can be proved and trying to prove it. When such a mathematician tries to evaluate a colleague's research, he asks himself, what new theorems are proved in this paper and what assumptions are relaxed and what is conjectured about what could be proved. The history of mathematics is limited to theorems proven by mathematicians. How they get to foresee these results before proving them is not part of history. The future of mathematics in this point of view is dedicated to precise conjectures which hopefully could be proved in their exact formulation as they are presented. Having such a vision about future is not accessible to any mathematician. Hard work and long time experience is needed for one to be able to foresee theorems which are not easily accessible yet to be proved. For these mathematicians, theorems are of central importance, not objects or concepts and the proof need not to be constructive or even illuminative. This resembles how a graduate students thinks about doing mathematics.

4) Problem Solving

What a professional researcher cares for is to formulate and solve important problems. Such a mathematician is concerned with the question that what are important problems in mathematics to be solved. Doing mathematics is nothing but formulating problems and solving problems. Learning mathematics is learning the achievements of mathematicians in proposing and solving important problems. Teaching mathematics is teaching the skill of problem solving or even teaching the skill of proposing interesting problems. Teaching how to evaluate importance of a problem, and how to find new tricks to progress in solving old problems. I have to confess that physicists are trained to get to this level of maturity faster than mathematicians. If such a mathematician wants to evaluate a colleague's research, he asks himself questions like; what problem this paper is trying to solve and what is the contribution of the author to solution of this problem? If this problem has been solved by a new technique? What are the abilities of this new technique in solving similar problems? How important is the problem and how difficult it is to settle it completely and how many years it may occupy mathematicians? Will they be able to have constant progress , so that they do not give up the problem and so on. History of mathematics, is history of achievements of mathematicians in proposing and solving problems. These mathematicians try to influence future of mathematics by proposing new problems to be solved and by arguing why these problems are important.

5) Theorization

In boundries of sciences, people care for paving the way for problem solver mathematicians by theorization of new mathematics to advance the techniques of problem solving and introduce more efficient tools for computing invariants and grasping new concepts. They try to reformulate the old mathematics and have the same achievements in the new reformulation proposed by them. Learning mathematics is learning the existing working theories and teaching mathematics is teaching the skill of theorization. Teaching the purpose of theorizations and how to evaluate theorizations. Doing research in its most sophisticated form is theorizing new theories and finding new formulations which better fit the goal of problem solvers. Such a mathematician is a leader and evaluates a research work by how it paves the way for future progress. History of mathematics is history of reformulations of mathematics and study the achievementsof these reformulations. The future can be understood by challenges theorization faces and by more explicit formulation of these challenges. This is the farthest that can be seen by a leader mathematician in a particular branch of mathematics.

What we have been through is the perspectives towards mathematics from inside. There are other perspectives which are suggested if we look at mathematics from outside coming from cognitive sciences and philosophy of knowledge which we will be discussing now in more detail.

6) Cognitive approach

There are mathematicians whose motivation for doing mathematics is cognitive achievements. Meaning that, they do mathematics so that they improve their cognitive structure. Their goal is to become wiser and a better thinker, which is of course an internal motivation. These people do not perform research for the sake of contributing mathematics or communicating with mathematical reader, but they have a personal internal reason for doing mathematics. They do mathematics because it satisfies them cognitively. They feel personally enriched when doing mathematics. Doing mathematics means revolutionizing their cognitive system. It gives them new perspective to everything they know. Knowing everything in a fresh and new way. And they keep handling these revolutions in their cognitive system. Learning mathematics does very much the same thing as the research doesand teaching mathematics means touching other people's cognitive structures and guiding and educating this structure. You should first recognize the mental state of the students and then decide what to teach them and how to effect their cognitive structures, in which directions they should move and which cognitive path they should take. What kind of mathematics can effect their cognitive personality and how? Which mind is prepared for which kind of mathematics to learn from. How does mathematics is interpreted in terms of cognitive structures of students' mind?

For such a mathematician, doing research is discovering the unknown abilities of human cognition. What could we understand that we could not imagine before that? While evaluating research of a colleague, such a mathematician asks oneself, what is the contribution of this paper to human cognition. What new things a mathematical mind can understand after absorbing the new material? What are contributions of this research to human cognitive standards and cognitive language and to the scope of imagination? History of mathematics for such a mathematician is nothing but the history of cognitive achievements of human being and therefore he is interested in history of all branches of science and also history of humanities, because they all can be translated to history of cognitive achievements of human mind. Therefore, such a mathematician is not only interested in understanding and learning mathematics, but he has wide interests and this wide income makes him have a wide outcome and he will be a philosopher of science and mathematics as well as a philosopher of education and even history. Philosophy of education, because of being engaged in education and teaching students , and philosophy of history because of having personal perspectives towards history of mathematics and sciences. A cognitive mathematicians view about the future of mathematics are formulated in forms of discovering unexplored parts of human cognition. One wants to mathematically explore parts of cognition structure which is discovered by other fields of science and humanities.

7) Incarnation of truth and unity of mathematics

The motivation of such a mathematician comes from a very Platonic perspective, believing in higher truth which is incarnating in mathematics and also in other sciences. That is why mathematics is such a successful tool in formulating a language for other sciences. In this philosophy, not only the mathematics is unified, but also all human sciences are unified and should be understood from a single common perspective. Plato's philosophy has been under constant change and reform through history and there are modern versions of this philosophy available to be used as philosophical background to this perspective towards mathematics.

In this perspective, learning mathematics is never ending efforts to understand what the original truth is which has been incarnated in mathematics. This needs the art of interpretation which is more accurately explained by the term “ta'vil” in Islamic philosophy, meaning to bring something to its original version. Teaching mathematics is in fact teaching the art of interpretation to the student. This goes back even beyond Plato, to the teachings of Paythaguras. If such a researcher wants to evaluate the research of a colleague, he tries to evaluate how good the mirror is author's mind to the truth incarnating in it and how clearly he can see the material which is presented to himand how able he is in the art of interpretation.

History of mathematics and history of science as well are history of achievements of human mind in understanding the higher more abstract truth. In this point of view, human mind is a discoverer and explorer of the world in which the higher truth lives in, and is a creator and originator of the incarnations of this truth to the language of human mind. Future of mathematics and other sciences is nothing but the future efforts of human to grasp the truth and this is more or less a function of the nature of truth incarnating and the abilities of human mind in grasping abstract material.

8) A Hierarchy on practical philosophies

We believe there is an order on the practical motivations introduced in previous chapters, and a mathematician goes through a path of perfection practicing mathematics according to these philosophies one by one. The hierarchy we have in mind is the order these philosophies are presented. We believe, the higher the philosophies of a practitioner is in this hierarchy, the more wise a mathematician he is. The clock of maturity goes dead in every person in some stage of their progress and they stop becoming more philosophically mature in some stage of their scientific life. This is the time when a researcher should step down from advantages given to him by the scientific community and leave room for youngesters to proceed.