Field-dependent and Field-independent Mathematics

Arash Rastegar

Institute for Advanced Study, Princeton, USA.
Sharif University of Technology, Tehran, Iran

Abstract: Being field-dependent and field-independent are two cognitive types in educational psychology. Mathematicians have field-dependent or independent cognitive structures produce mathematics which is dependent on development of other fields or independence of such developments. Oddly, it is the case that most strong learners are field-independent but more strong mathematicians do mathematics in field-dependent style.

Introduction

It is not necessarily the case that mathematicians who are field-dependent learners do mathematics in the field-dependent style or that mathematicians who are field-independent learners do mathematics in the field-independent style, but oddly it is the case that being field-dependent and independent make sense also for branches of mathematics,as if developments of a branch of mathematics is analogous to human learning and it satisfies analogous psychological classifications as human learning. usually mathematicians doing field-independent mathematics are "frog" according to Freeman Dyson and mathematicians doing field-dependent mathematics are "birds"; the former being analytic and the latter being holistic and encyclopedic. It is not necessarily the case that each branch of mathematics is fully field-dependent or fully field-independent. There could be a mathematician doing field-dependent style of research in a branch of mathematics which has field-independent style of development and vice versa.

Field-dependent cognitive Style

Cognitive styles in educational psychology are usually treated as cognitive styles of learning, but for us it is a property of the cognitive structures and in other words a cognitive style of thinking. Defining field-independent style of learning it is said that some students learn very much depending on the atmosphere that they are being taught in or depending on what is going on in the classroom or depending on the material being taught or rely on the information provided by outer world, the field or frame of a situation and their cognition towards other things. This concept was first proposed by Herman Witkin the American psychologist in 1962. field-dependent people tend to work better in teams as they tend to be better at interpersonal relationships. They prefer teaching in the styles that allow interaction and discussion between the students and teachers. But field-independent learners have tendency to separate details from surrounding context. In a field-dependent mode in individual pattern recognition is strongly dominated by the holistic organization of the total perceptual field with it parts being perceived as "fused". In contrast, in the field-independent mode of perceiving, the individual is more likely to see the parts of the field as distinct from the organized ground. Field-independent individuals when compared to field dependence ones they are more capable of restructuring the perceptual field or imposing a structure if one is learning. They also tend to act more autonomously than field-dependent.

Field-independent cognitive Style

There is a relationship between the strength of field independence and problem solving performance. The solution depends on the individual using a critical element in a different context form the one in which had originally been presented, thus showing a connection between analytical and structuring abilities. Field-dependent individual are more strongly influenced by the immediate context. It is not the case that field-independent mathematicians necessarily do mathematics in the field-independent style. The concepts of field-dependent and field-independent learners is completely a different concept from field-dependent and field-independent branches of mathematics. Field-dependence branches of mathematics have tendency to interact with other branches which is different from field-independent branches which tend to answer philosophical questions raised inside the field rather than analogues with what happens in other fields of mathematics. Both of these two type of mathematics can be performed by field-dependent learners and field-independent learners. In better awards, this is not analogous to analytic and holistic branches of mathematics respectively. Analogies in field-independent branches are among sub-fields of the same branch and those in field-dependent branches are among this branch and other branches of mathematics. For example number theory and algebraic geometry are holistic and therefore very much depends on analogies between these fields and other branches of mathematics.

Field-dependent and independent style of doing mathematics

Field-dependent style means doing mathematics depending on research in other fields. This dependence is by borrowing ideas from them or trying to solve their problems using this field's methods or trying to solve one's field problems using methods of other branches of mathematics. This is of course a very natural thing to do. But there is another style of doing mathematics in which the mathematician fixes the methods of attacking the problem before facing the problem or looks for problems which can be solved by predetermined tools. This way of doing mathematics is called field-independent to style. Number theory and algebraic geometry and other holistic fields of research tend to be field-dependent. They try to borrow ideas from different places in mathematics. Dictionaries and analogies play an important role in this style of doing mathematics. Analogies could be analogies between problems or analogies between theories or cognitive analogies. We have discussed the differences in an earlier paper. For field-independent Style there is no room for analogies and dictionaries. Every field-dependent branch of mathematics lean on its own methods of calculation and problem-solving and it makes itself busy with answering philosophical question raised but its own theories. Such a branch of mathematics tries to forget its sources of existence outside itself and tries to keep itself alive by keeping its local reasons of existence at each stage of development.

How to influence work of other mathematicians in other branches

Field-dependent branches of mathematics not only get influenced by other branches of mathematics but also they try to influence other branches as a result of this interaction. So interaction between difference benches of mathematics is by influencing and becoming influenced. There are different methods of influencing other branches of mathematics. Posing problems in other branches analogous to problems in the influencing branch is one way of influencing. The other method of influence is finding analogies between theories in two branches of mathematics. Finding cognitive analogies is the deepest and most efficient way of influencing other branches of research performed in a given branch of mathematics. Note that the concept of a branch of mathematics influencing another branch is a totally different concept than in mathematician influencing another mathematician. Former is about influence of society on another society but the latter is about influence of an individual on another individual. When a branch is influenced by another branch it is still trying to satisfy its own goals but under the influence of the other branch. In personal influence even the goals can be influenced. In branches influencing each other the goals define each branch and cannot be influenced. This is a point that should be kept in mind while researchers in one field try to influence researchers in other fields of research. They should not mess with other goals.

Influence by posing problems

Trying to give examples of problems from one field to another field of research. For example trying to determine a group giving information about its representations is a problem posed by group Theory to many other branches of mathematics. But analogy between group representation and manifold embedding in Euclidean space is a higher influence. In fact it is a cognitive influence. Sometimes concepts are the means of influence from one field to another. For example, similarity of finite groups and compact groups in representation theory is a theorizing analogy which allows to prove similar results to finite group Representations in representation theory of compact groups. Another example is the problem of projectivization of Euclidean space which leads to compactification of topological spaces and compactification of moduli spaces. Ideas going from one place to another place in mathematics, could be in the form of problems, tricks, formal ideas, analogies, and many other means. We do not intend to classify such influences. The problem of finding maximum and minimum of a function lead to calculus variations and discrete optimization theory in continuous and discrete mathematics. The problem of defining (co)homology theory is a problem posed by algebraic topology to manyother fields of the mathematics. The problem of calculating integral of the function leads to trying to define a measure in many different unrelated fields of mathematics. A higher way to influence other branches of mathematics is finding theoretical analogies.

Influenced by developing analogies between theories

There are many examples of theoretical analogies which have influenced interaction between different branches of mathematics. Analogy between number field and function field relates algebraic geometry and number theory. Vojta conjectures Relates Diophantine Theory with complex analysis. Jullivan's dictionary relates Julia sets to limit sets of Kleinian groups. Arakelov theory is based on analogy with intersection theory on compact varieties. In analogies, a one-to-one correspondence between important concepts on the two sides and also between concept relations is found. Sometimes the analogies are so deep that they get accepted like a philosophical postulate. Such a strong and deep theme could be seen in the analogy between number fields and function fields. Several sub-analogies are found in this context many of which could hardly be united to single analogy. Some of them work for function fields of one variable over finite field and some work for general functions field over arbitrary field. Sometimes the analogies go further than that. One finds a conjectural one-to-one correspondence between objects which seem to carry the information through a bridge from one field of mathematics to the other field. Such analogies are called "Cognitive Analogies" and they encompass a much deeper relationship between two branches of mathematics. A major example of such a cognitive analogy is given in the framework of Langlands program.

Influence by finding cognitive analogy

Nobody knows why motives should be modular. Algebraic varieties, L-functions, several equivalent formulations of modular forms, Galois representations, all contain objects which behave similarly as if there is a one-to-one correspondence between the worlds, if one assumes certain restrictions. Experts in Langlands

Program can tell us a great deal how each of these languages influence the others because of the cognitive analogy between them. In special cases, this one-to-one correspondence is constructed and proved. An important example is the proof of Shimura-Tanyama conjecture in general case by Wiles and collaborators. Although Wiles results does not shed light on why motives should be modular, but it has several implications on development of the theory of elliptic curves, modular forms, and Galois representations. The same methods lead to tackle several important conjectures in number theory namely, Serre's conjecture, Sato-Tate conjecture and others. This shows that the method of finding cognitive analogies is by far stronger than finding theoretical analogies and problem analogies in development of mathematics. This is an important reason to support the philosophy of mathematics and cognitive mathematicians to do mathematics. Motivation of cognitive mathematician to do mathematics is having cognitive improvement. This can be thought of, as a philosophy of education in general and particularly a philosophy of mathematics education. Regarding This philosophy of mathematics, the reader is referred to other papers of the author on this topic.

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